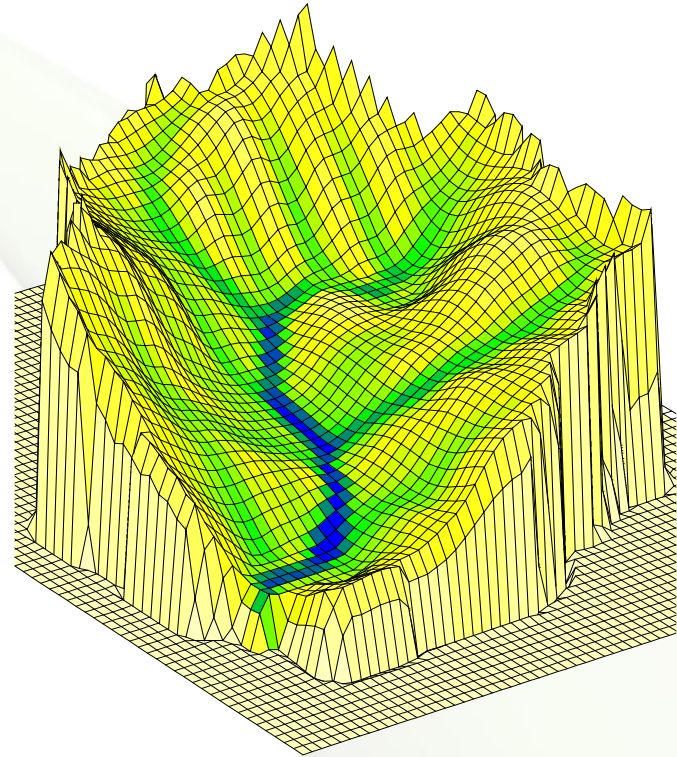


유역지형의 복잡계 구조를 이용한 산사태 우심지역 식별 방법

JOO-CHEOL KIM

Contents

- **Objective and Overview**
- **Power Law Distribution**
- **Methodology**
- **Case Study**
- **Conclusion**



Objective and Overview

- **Main Objective is to suggest methodology for identifying vulnerable points in the basin to soil losses/disasters**
- **By using the relation between local slope and drainage area based on power law distribution.**
- **Geomorphologic factors extracted from GIS are used to represent shear stress and stream power as well as drainage area**
- **And the method of maximum likelihood is applied for parameter estimation of power law distribution.**
- **Within the framework of power law distribution**
- **To analyze mass aggregation structure and energy expenditure pattern within the river basin**
- **And to test the methodology to identify vulnerable region in the basin susceptible to soil losses/disasters**

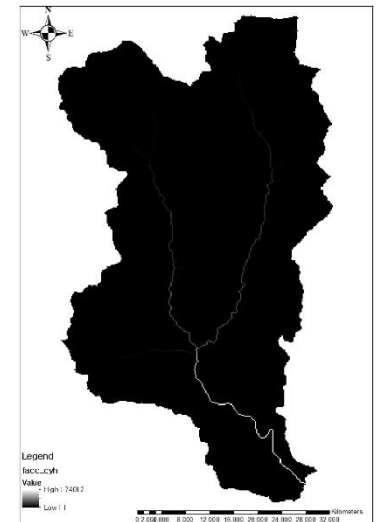
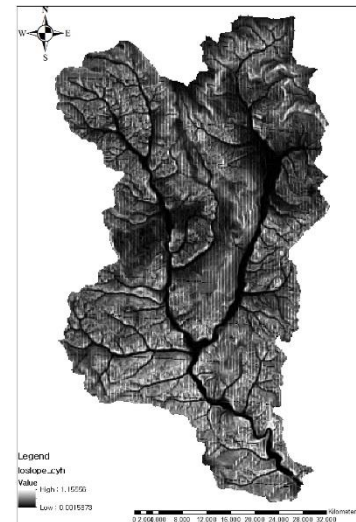
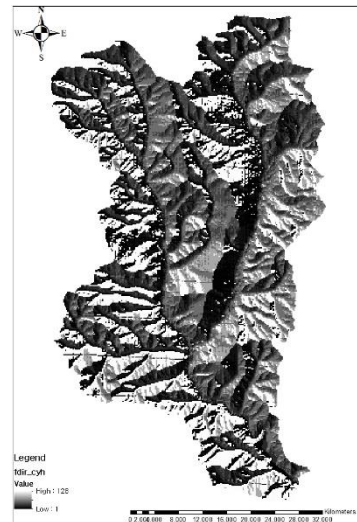
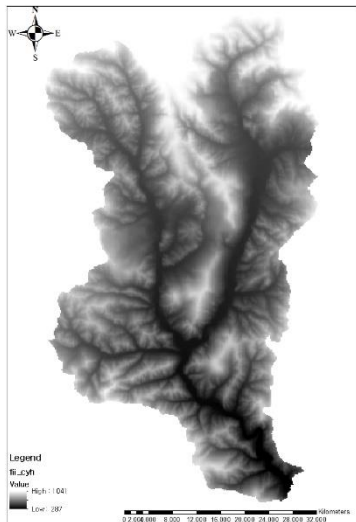
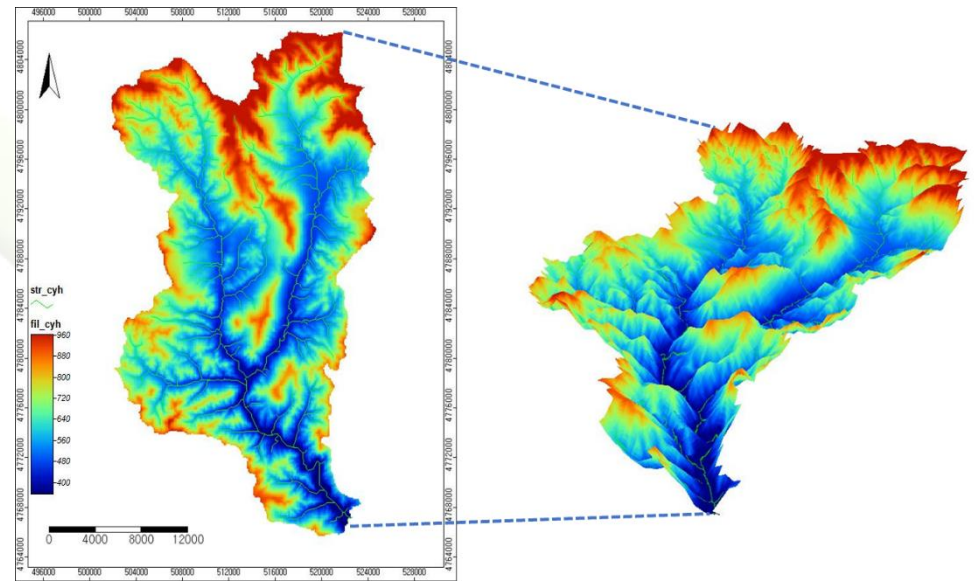
Study Site: 朝阳河流域 (吉林省 邊朝鮮族自治州)

■ Choyang Creek Basin

➤ Drainage Area: 599 km²

■ DEM Analysis

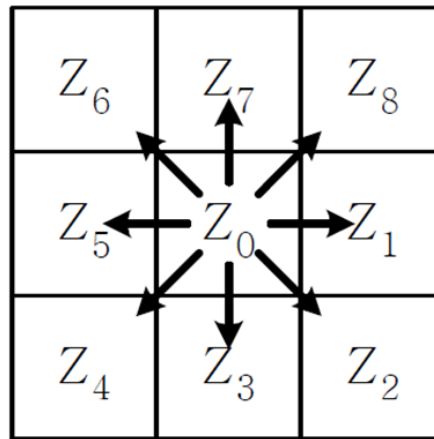
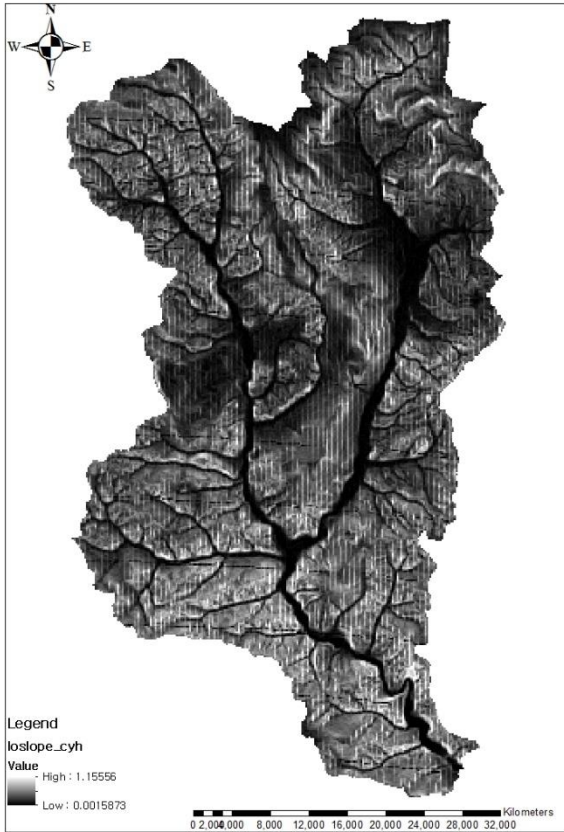
- SRTM DEM
- Grid Resolution : 90 m
- GIS tool: TauDEM (Tarboton)



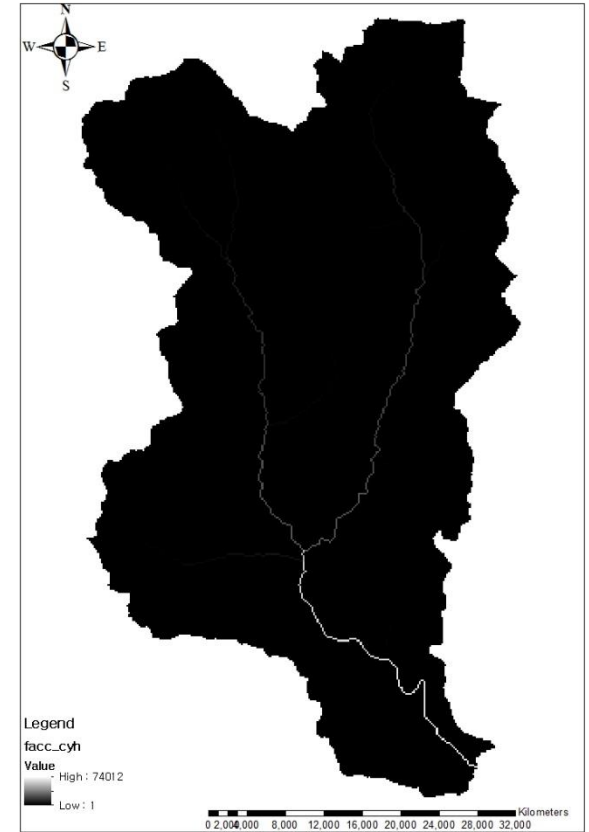
Study Site

◆ 8 Flow direction method

Local Slope



Drainage Area



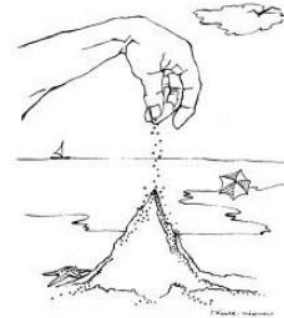
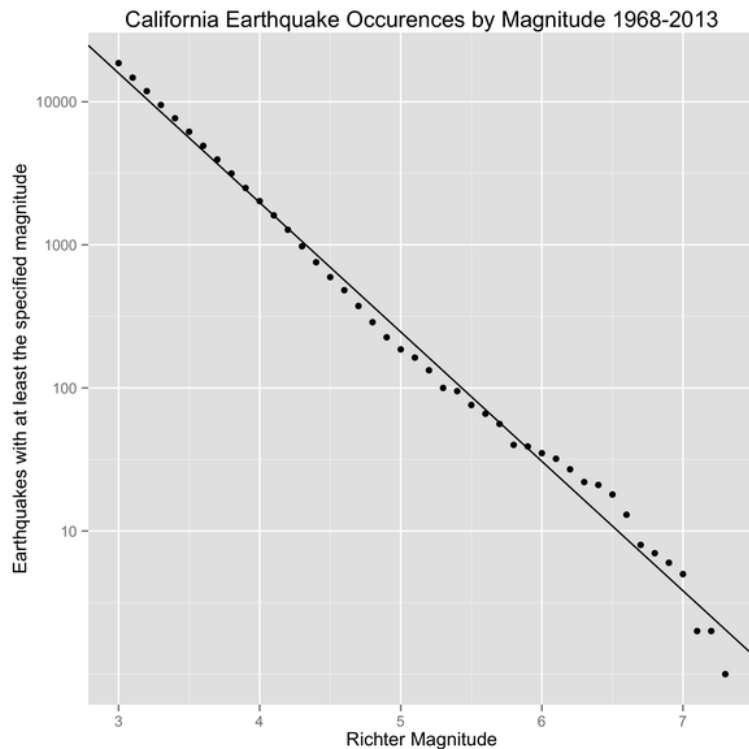
$$S_j = \frac{Z_0 - Z_j}{L}; \quad j = 1, 2, \dots, 8$$

$$A_i = \sum_{j \in nn(i)} W_{ij} A_j + 1$$

Power Law Distribution

- **Power Law Distribution is the main feature of complex system presenting in various fields**
 - Gutenberg-Richter Law: Geophysics
 - Pareto Law: Economy
 - Zip Law: Linguistics

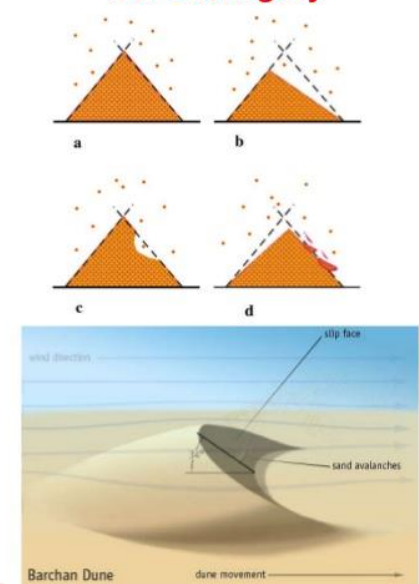
$$p(x) \propto x^{-\alpha}$$



Per Bak, Danish theoretical physicist, 1948 – 2002. Co-author with Chao Tang, Kurt Wiesenfeld of the new paradigm of "Self-organized criticality" via sandpiles.

Mathematics Awareness Month, April 2011.

Sand-piles model of Complexity Per Bak legacy



Power Law Distribution

- **Runoff Aggregation System (Rodriguez-Iturbe, 1992)**

- ❖ 1-D fractional Brownian Motion

$$p(x, L_{\parallel} | x_0) = \frac{1}{\sqrt{4\pi D_H L_{\parallel}}} e^{-\frac{(x-x_0)^2}{4D_H L_{\parallel}}} = \frac{1}{\sqrt{4\pi D L_{\parallel}^{2H}}} e^{-\frac{(x-x_0)^2}{4D L_{\parallel}^{2H}}}$$

Hurst Exponent

fractional Brownian Diffusivity $D_H (= D L_{\parallel}^{2H-1})$



$$f(L_{\parallel}) \propto L_{\parallel}^{-(H+1)}$$

$$A \propto WL \propto L_{\parallel}^{H+1}$$



$$f(A) \propto A^{-\left(\frac{2H+1}{H+1}\right)}$$

$$F(A) \propto A^{-\left(\frac{H}{H+1}\right)}$$



➤ **Drainage area has the same distribution as earthquake**

$$p(x) \propto x^{-\alpha}$$

Power Law Distribution

▪ Generalization (Newman, 2005)

❖ Mathematical Formulation of Power Law Distribution

$$p(x) \propto x^{-\alpha} \Rightarrow f(x) = Cx^{-\alpha} \Rightarrow x_{\min} < x < \infty$$

$$\therefore f(x) = \left(\frac{\alpha - 1}{x_{\min}^{-\alpha+1}} \right) x^{-\alpha} = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}} \right)^{-\alpha}$$

$$F(x) = \left(\frac{x}{x_{\min}} \right)^{-(\alpha-1)}$$

$$E[x^l] = \int_{x_{\min}}^{\infty} x^l f(x) dx = \begin{cases} \infty & ; \alpha < l + 1 \\ \left(\frac{\alpha - 1}{\alpha - 1 - l} \right) x_{\min}^l & ; \alpha > l + 1 \end{cases}$$

➤ ***If exponent of power law distribution has the value less than 2 statistical moments cannot be defined for all orders.***

➤ ***Therefore power law distribution is a kind of signature for scale invariance.***

Methodology

▪ Definition of Geomorphologic Factors based on DEM

$$I = k Q^{\zeta} S^{\eta}$$

➤ Discharge

$$Q_i \propto A_i$$

➤ Shear Stress

$$\tau = \rho_w g R S = C_f \rho_w v^2$$

$$\tau \propto R S$$

$$\tau_i \propto A_i^b S_i$$

$$R (= Wd / (W + 2d))$$

$$R \propto d$$

❖ Hydraulic Geometry

$$W = c_1 Q^e; \quad d = c_2 Q^b; \quad v = c_3 Q^f$$

➤ Stream Power

$$P \propto Q S$$

$$P_i \propto A_i S_i$$

Methodology

▪ Power Law Distribution of Geomorphologic Factors

➤ Discharge $f(A) \propto A^{-\alpha_1}; \alpha_1 = \frac{2H+1}{H+1}$

❖ Scaling of Local Slope and Drainage area $S \propto A^{-\theta}$

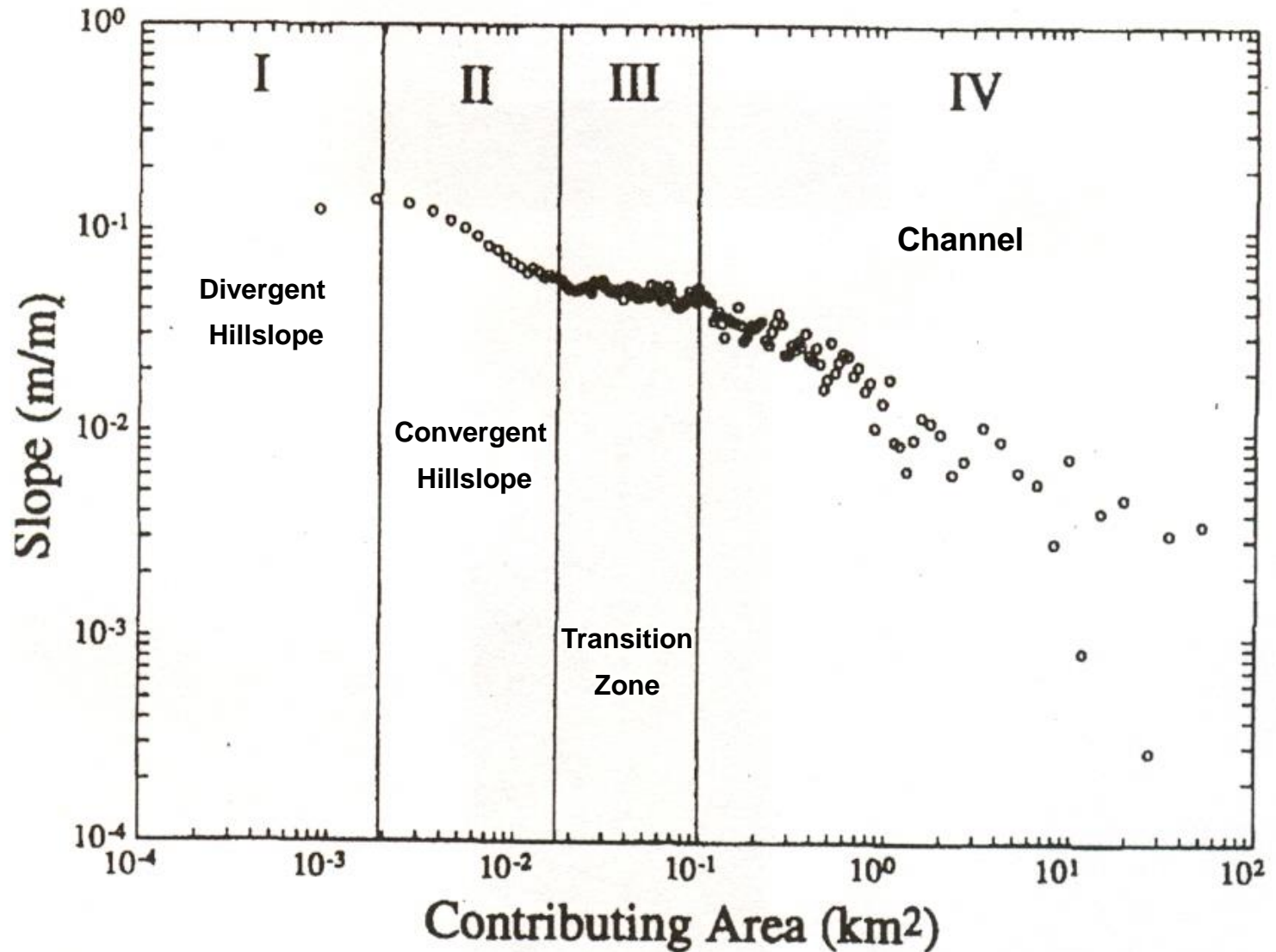
➤ Shear Stress $f(\tau) = f(A) \frac{dA}{d\tau} \propto \tau^{-\alpha_2}; \alpha_2 = \frac{1}{b-\theta} \left(\frac{2H+1}{H+1} + b - \theta - 1 \right)$

➤ Stream Power $f(P) = f(A) \frac{dA}{dE} \propto P^{-\alpha_3}; \alpha_3 = \frac{1}{1-\theta} \left(\frac{2H+1}{H+1} - \theta \right)$

This is equivalent to the formulation of fluvial process within the river basin as complex system with respect to Mass(Discharge), Force(Shear stress) and Energy(Stream Power) within the framework of Power Law Distribution

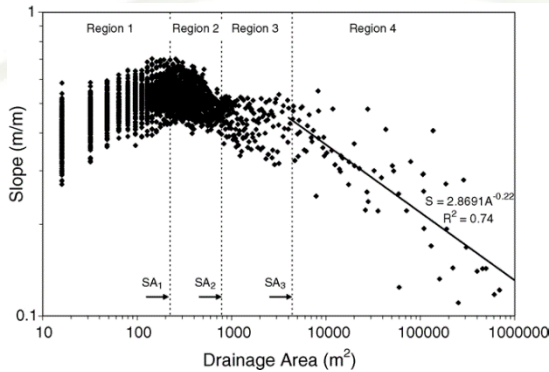
Methodology

Scaling Regimes of Local Slope and Drainage Area

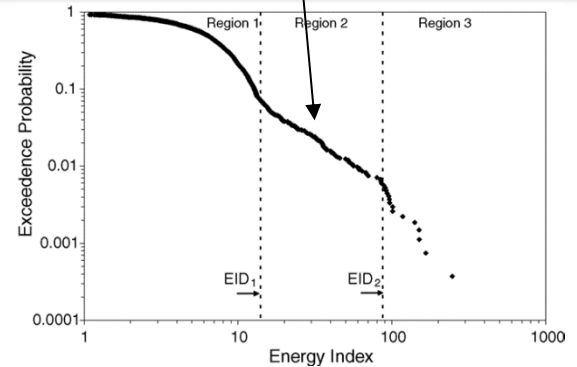


Methodology

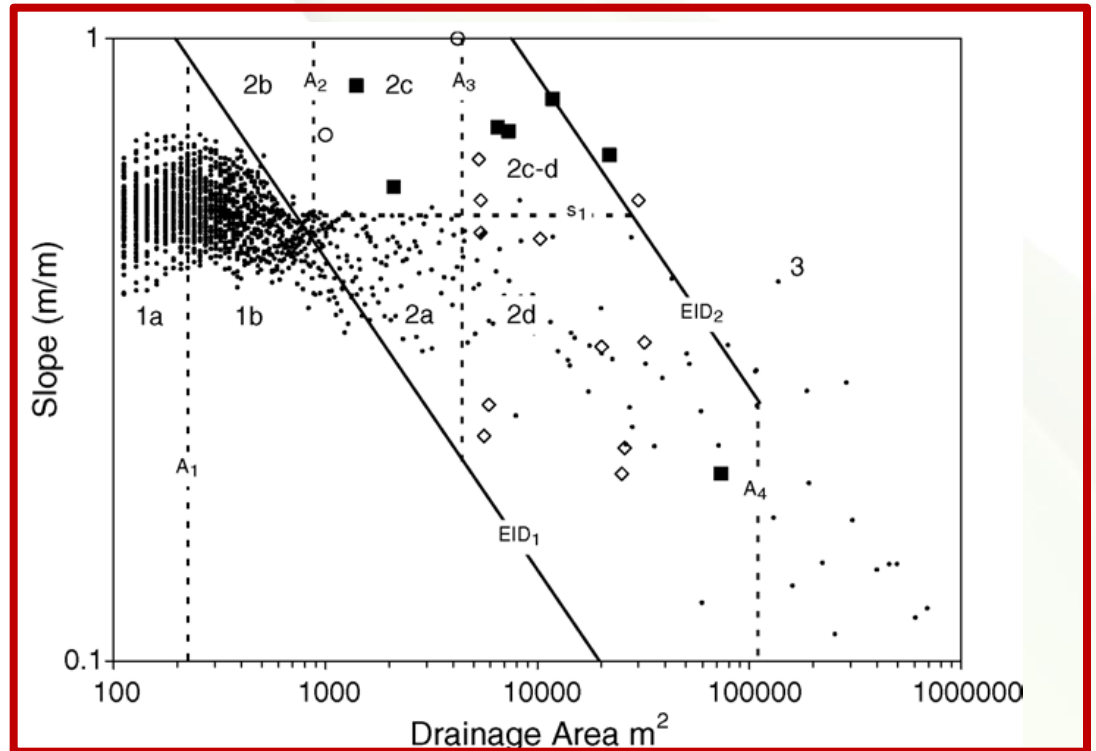
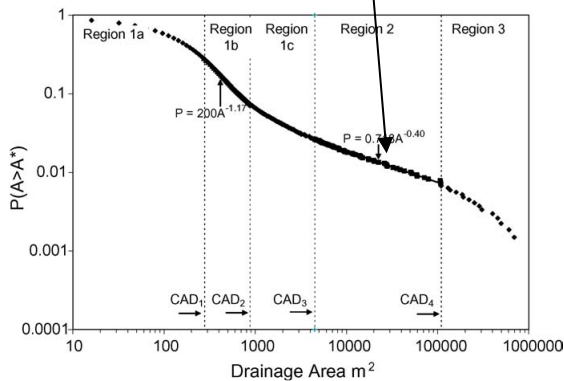
Classification of Local Slope and Drainage Area (McNamara, 2006)



$$f(\tau) = f(A) \frac{dA}{d\tau} \propto \tau^{-\alpha_2}$$



$$f(A) \propto A^{-\alpha_1}$$

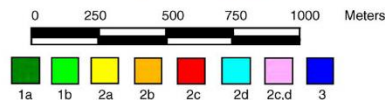
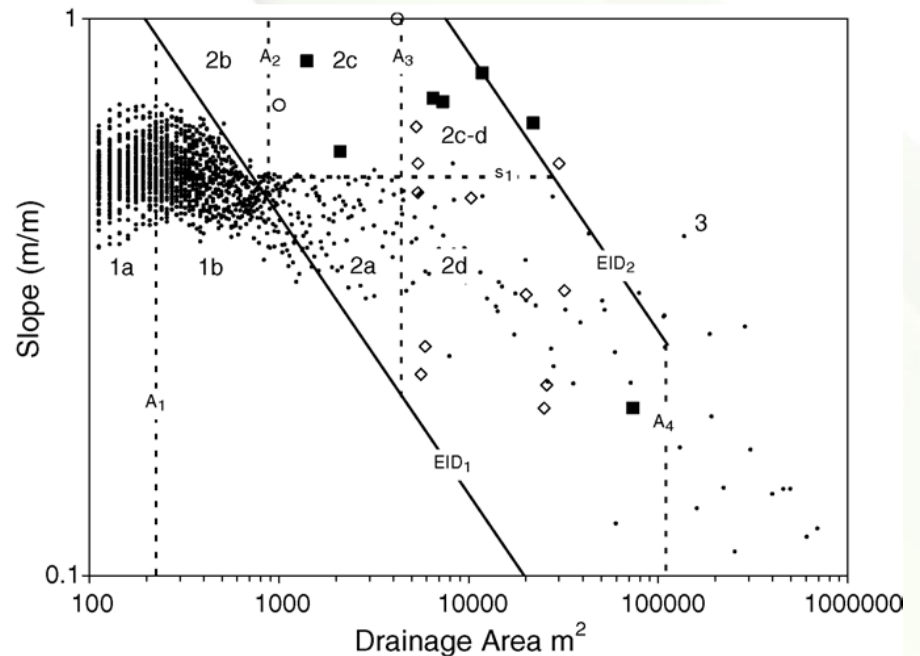
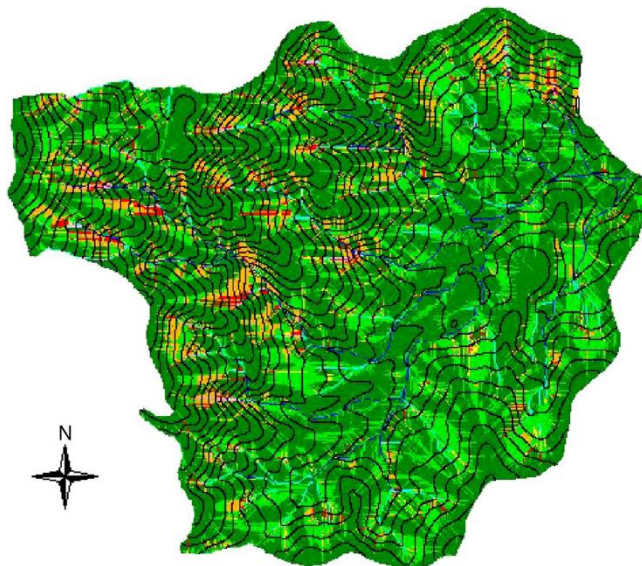


Methodology

Classification of Local Slope and Drainage Area (McNamara, 2006)

Stability Condition of Slope–Area Characteristic Regime Based on Channel Initiation

	Energy	Slope	Area	Stability
1a	deficit	-	-	convex hillslope
1b	deficit	-	-	concave valley
2a	enough	mild	small	stable
2b	enough	steep	small	landslide active
2c	enough	steep	large	overland flow active
2d	enough	mild	large	ground water active
3	overabundant	-	-	channel



Methodology

▪ Parameter Estimation of Power Law Distribution

➤ Likelihood

$$f(x) = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}} \right)^{-\alpha}$$



$$p(x | \alpha) = \prod_{i=1}^n \frac{\alpha - 1}{x_{\min}} \left(\frac{x_i}{x_{\min}} \right)^{-\alpha}$$

➤ Maximum Likelihood with assumption of x_{\min}

$$\partial \ln p(x | \alpha) / \partial \alpha = 0$$



$$\hat{\alpha} = 1 + n \left[\sum_{i=1}^n \ln \left(\frac{x_i}{x_{\min}} \right) \right]^{-1}$$

➤ Kolmogorov-Smirnov (KS) test

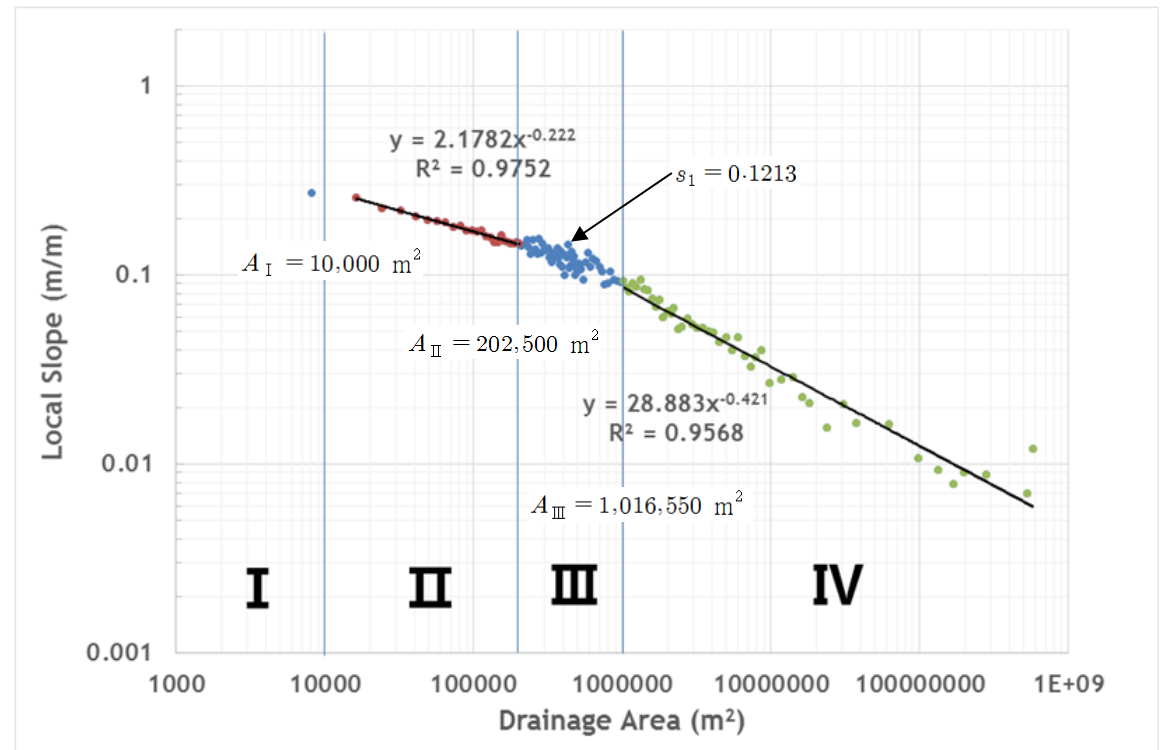
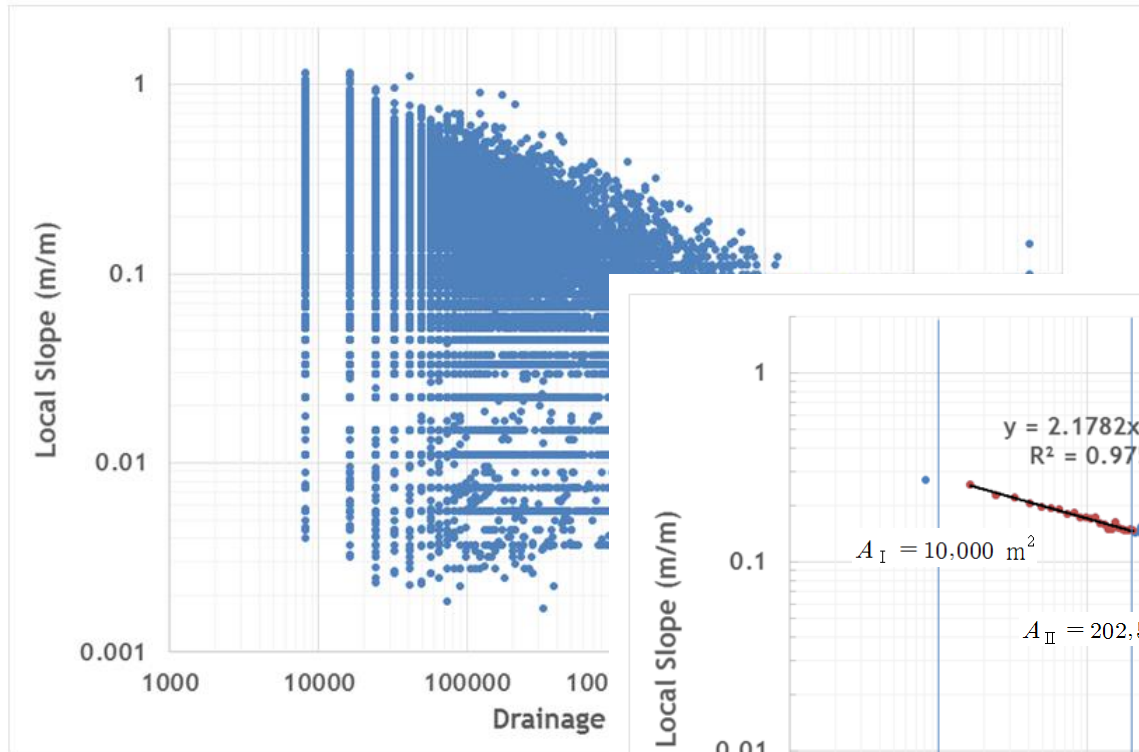
$$\min D = \max_{x \geq x_{\min}} |S(x) - F(x)|$$

$$S(x) = m/n + 1$$

$$F(x) = \left(\frac{x}{x_{\min}} \right)^{-(\alpha-1)}$$

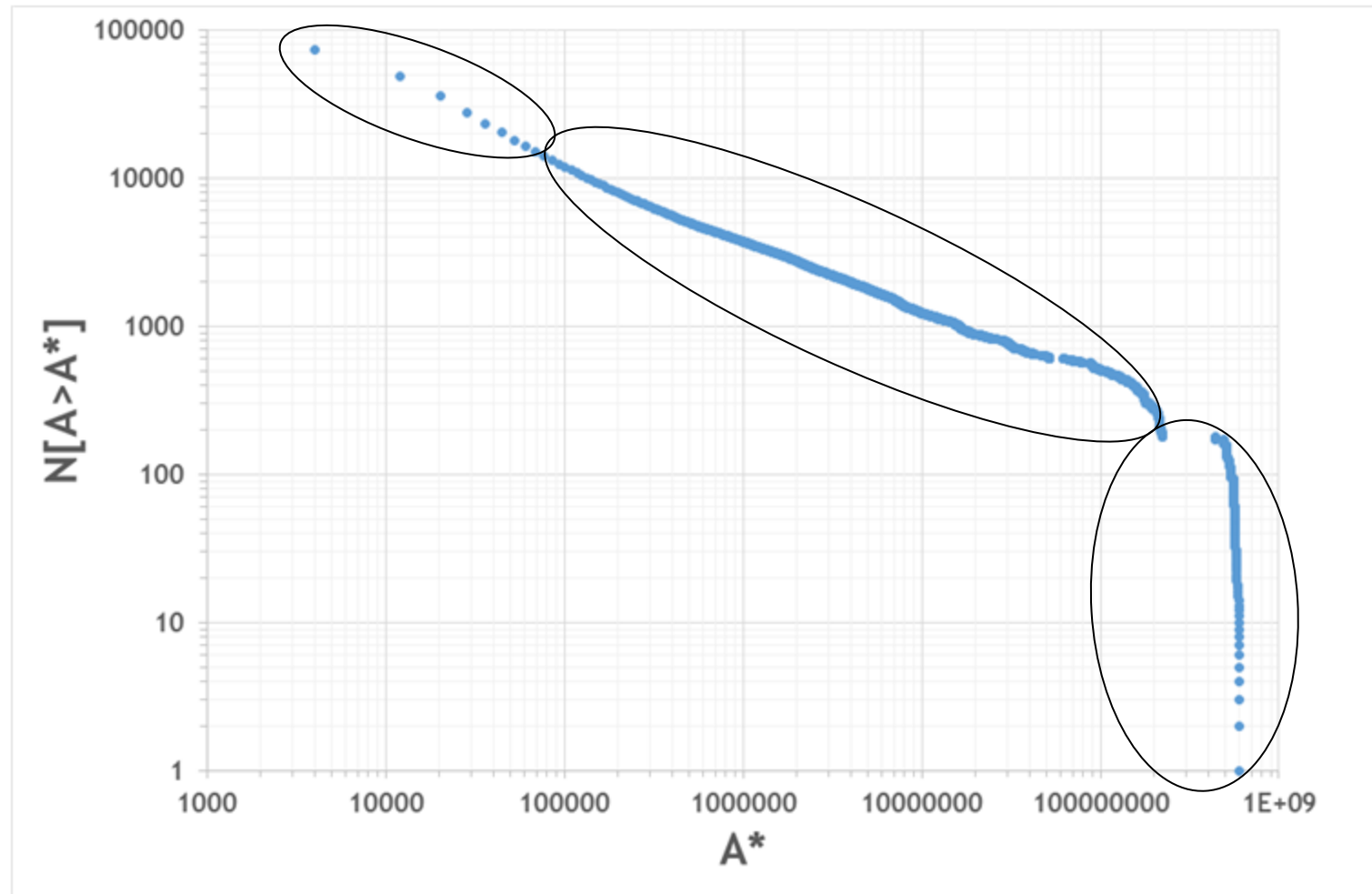
Case Study

▪ Relation between Local Slope and Drainage Area



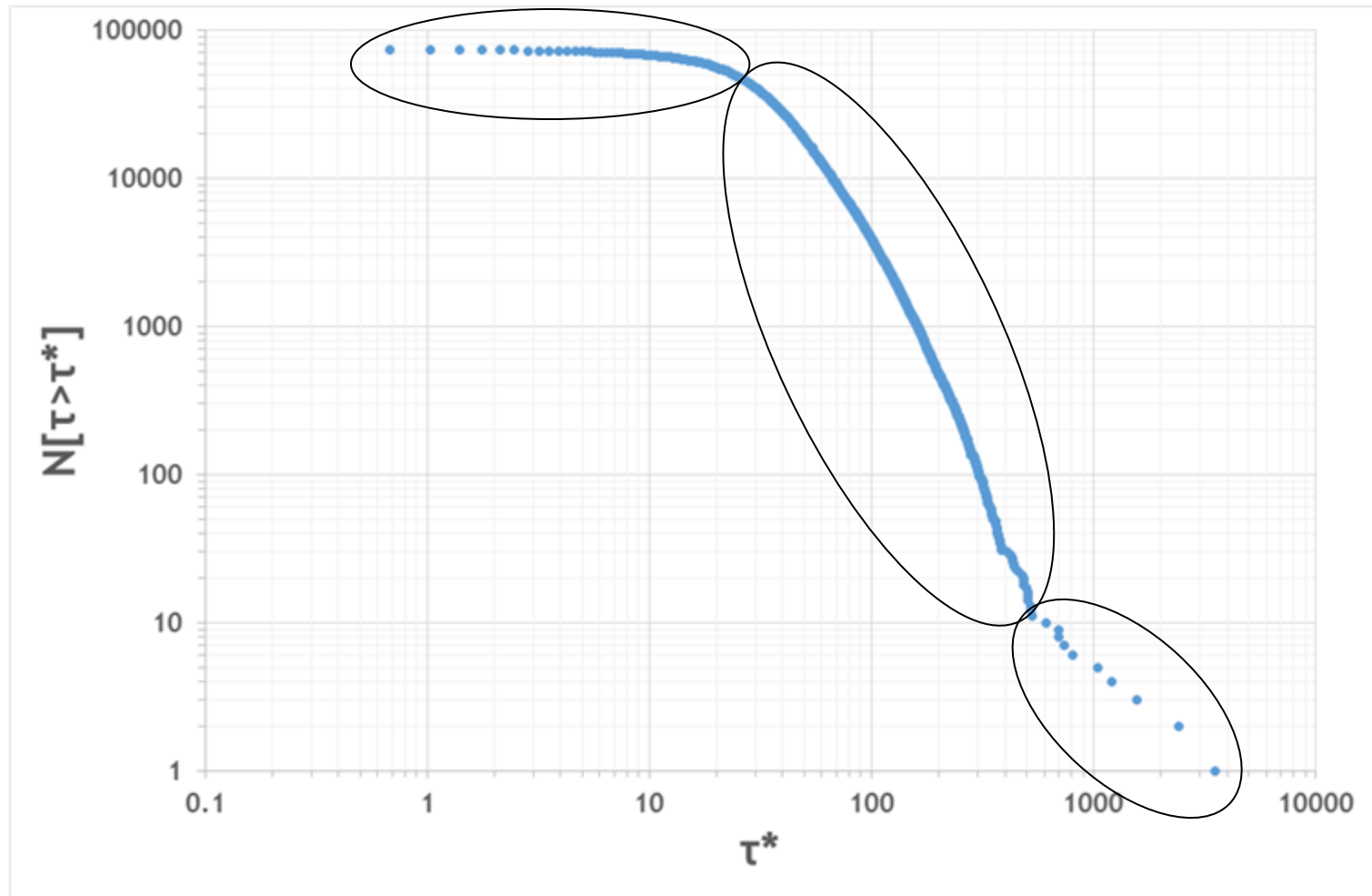
Case Study

- Complementary Cumulative Distribution of Drainage Area



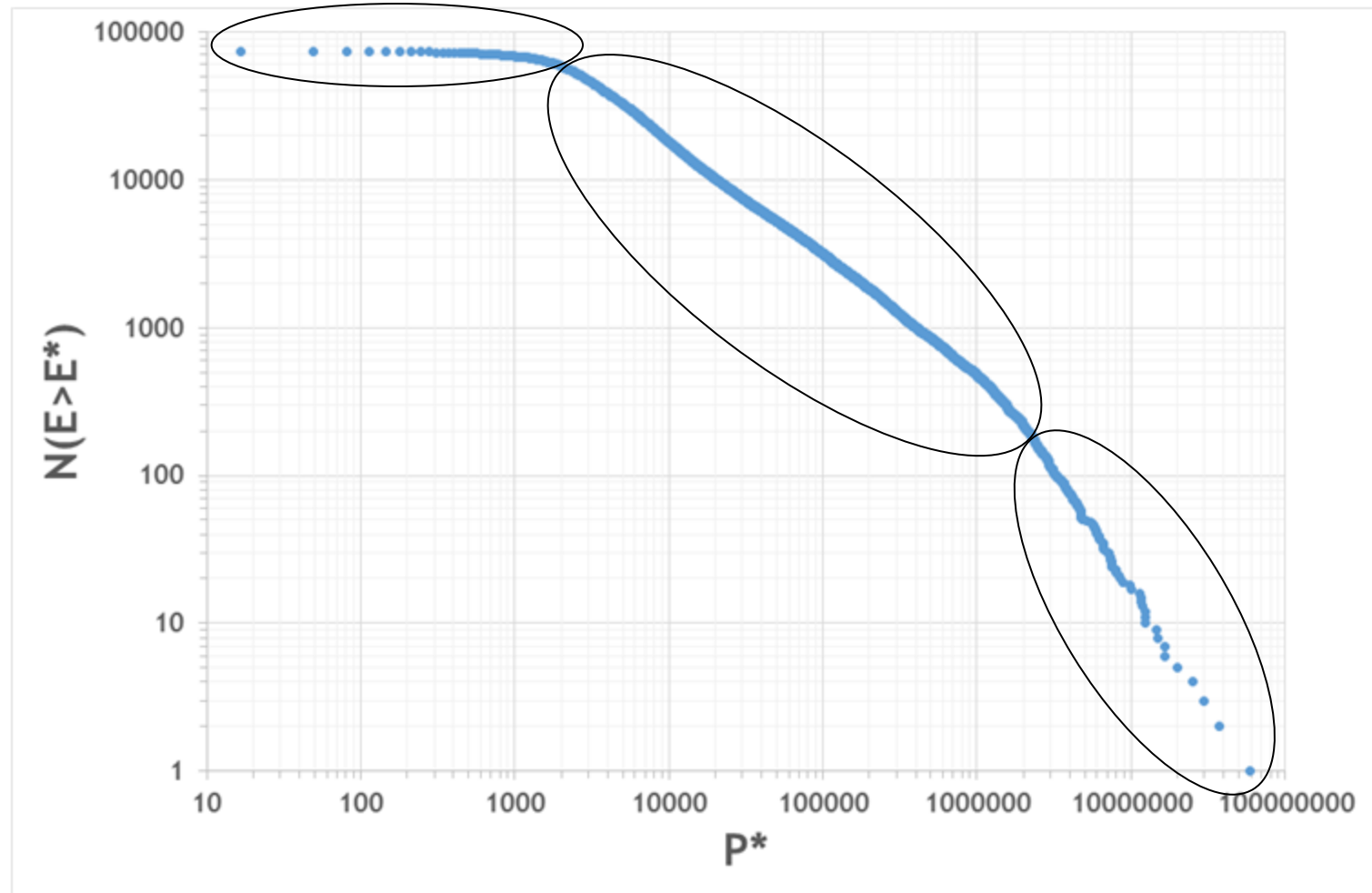
Case Study

▪ Complementary Cumulative Distribution of Shear Stress



Case Study

▪ Complementary Cumulative Distribution of Stream Power



Case Study

In Summary

- There are three different parts in the complementary cumulative distribution curves on log-log plot.
- ✓ ***Linear trend in the middle part, different trend in the upper part and finite scale effect in the tail part*** except shear stress.
- ❖ *So drainage area and stream power has scale invariant property.*
- In the case of shear stress there is no finite scale effect in the tail part.
- ❖ *Shear stress might be a scale dependence factor not following power law distribution*

Case Study

▪ Fitting Power Law Distribution by ML

Parameter Estimation of Power Law Distribution

I	α	x_{\min}
A	$\alpha_1 = 1.50$	$A_{\min} = 166050 \text{ m}^2$
τ	$\alpha_2 = 4.42$	$\tau_{\min} = 158.2 \text{ m}$
E	$\alpha_3 = 1.78$	$E_{\min} = 10838 \text{ m}^2$

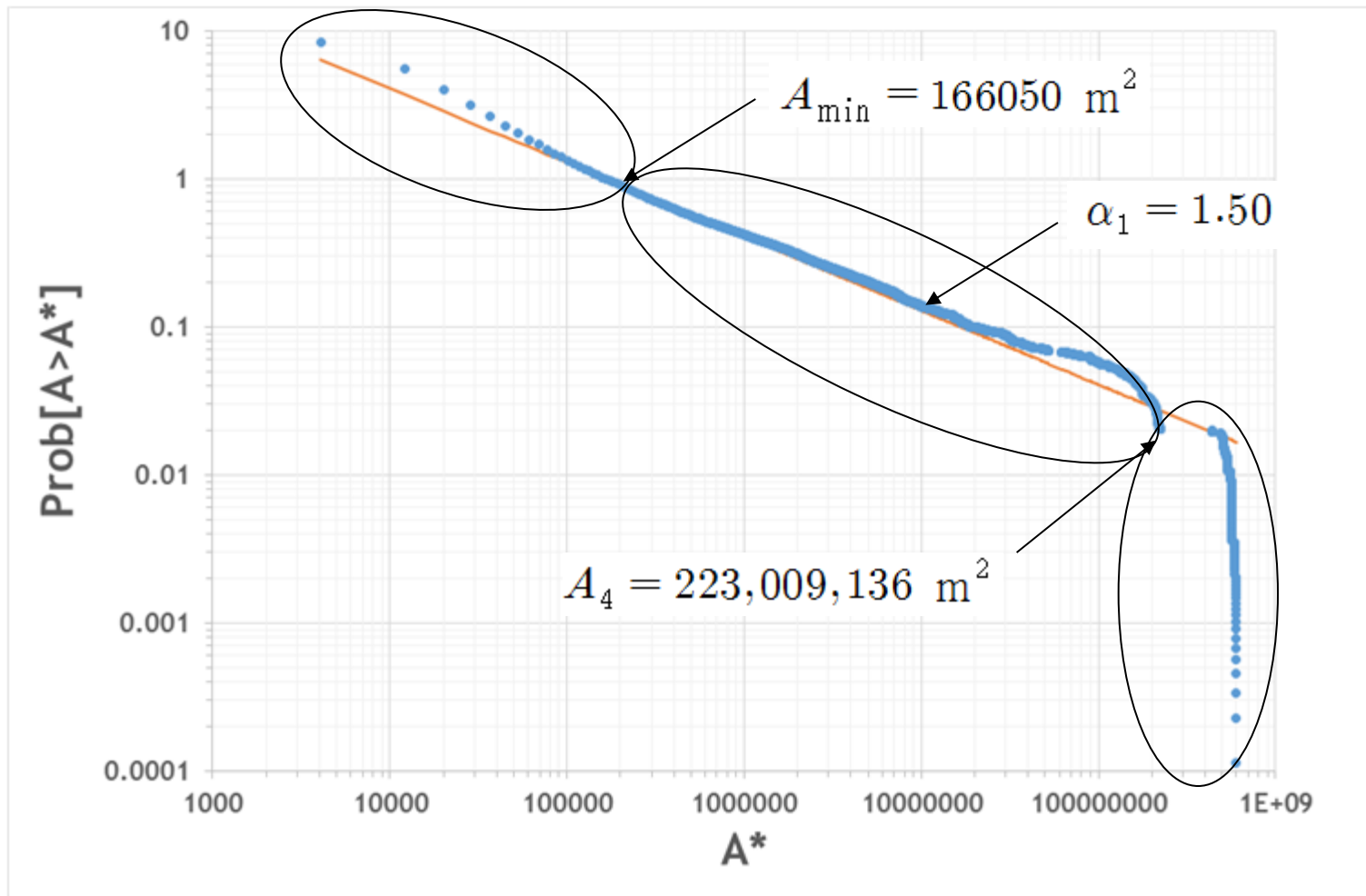
Statistical Moments of Tractive Force

$E[\tau] \text{ (m)}$	$E[\tau^2] \text{ (m}^2\text{)}$	$Var[\tau] \text{ (m}^2\text{)}$	Cv	$E[\tau^3] \text{ (m}^3\text{)}$	Sk
223.57	60276.87	20192.48	0.45	32240090.57	30.88

In the case of shear stress, first to third order of statistical moments can be defined meaning scale dependence property!!!

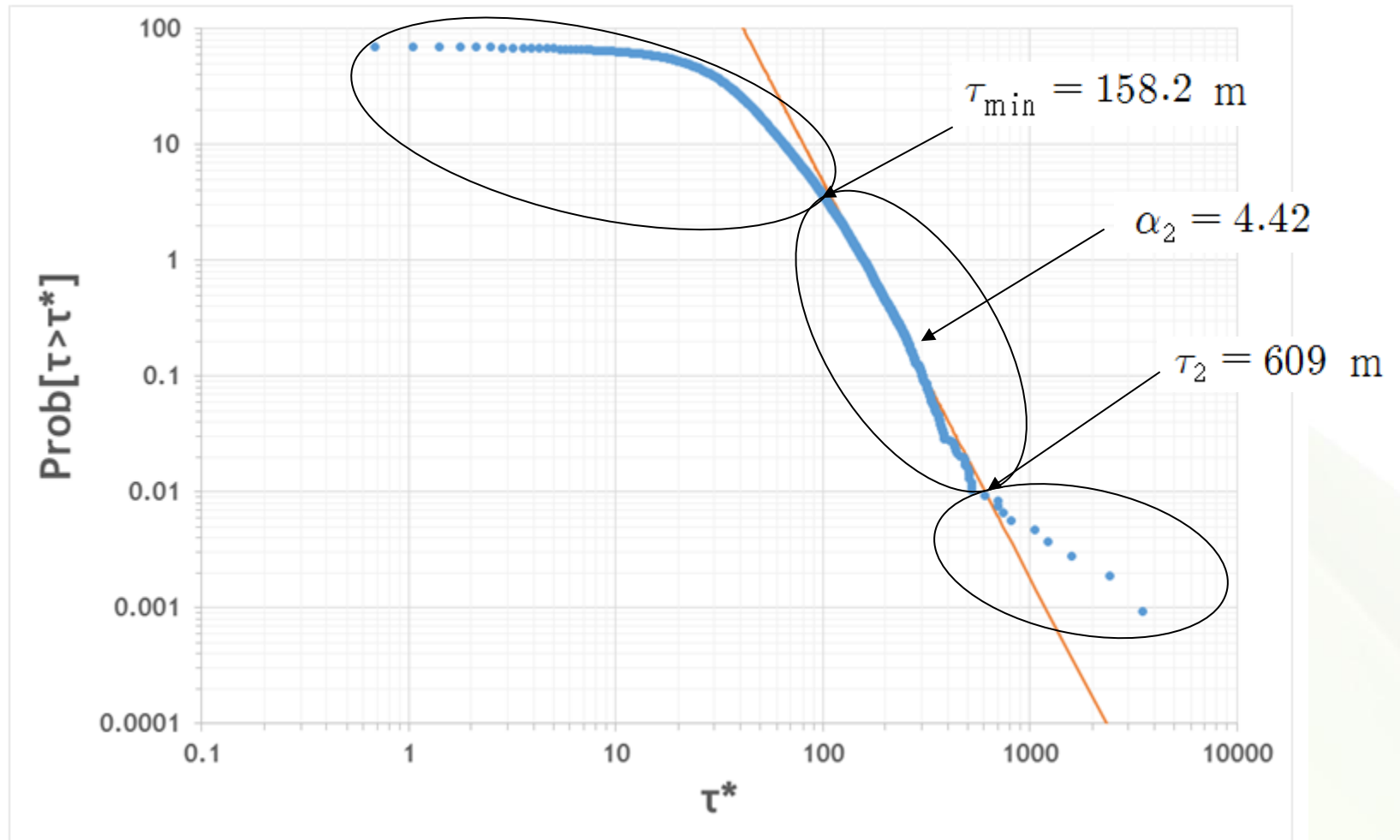
Case Study

▪ Fitting Power Law Distribution by ML



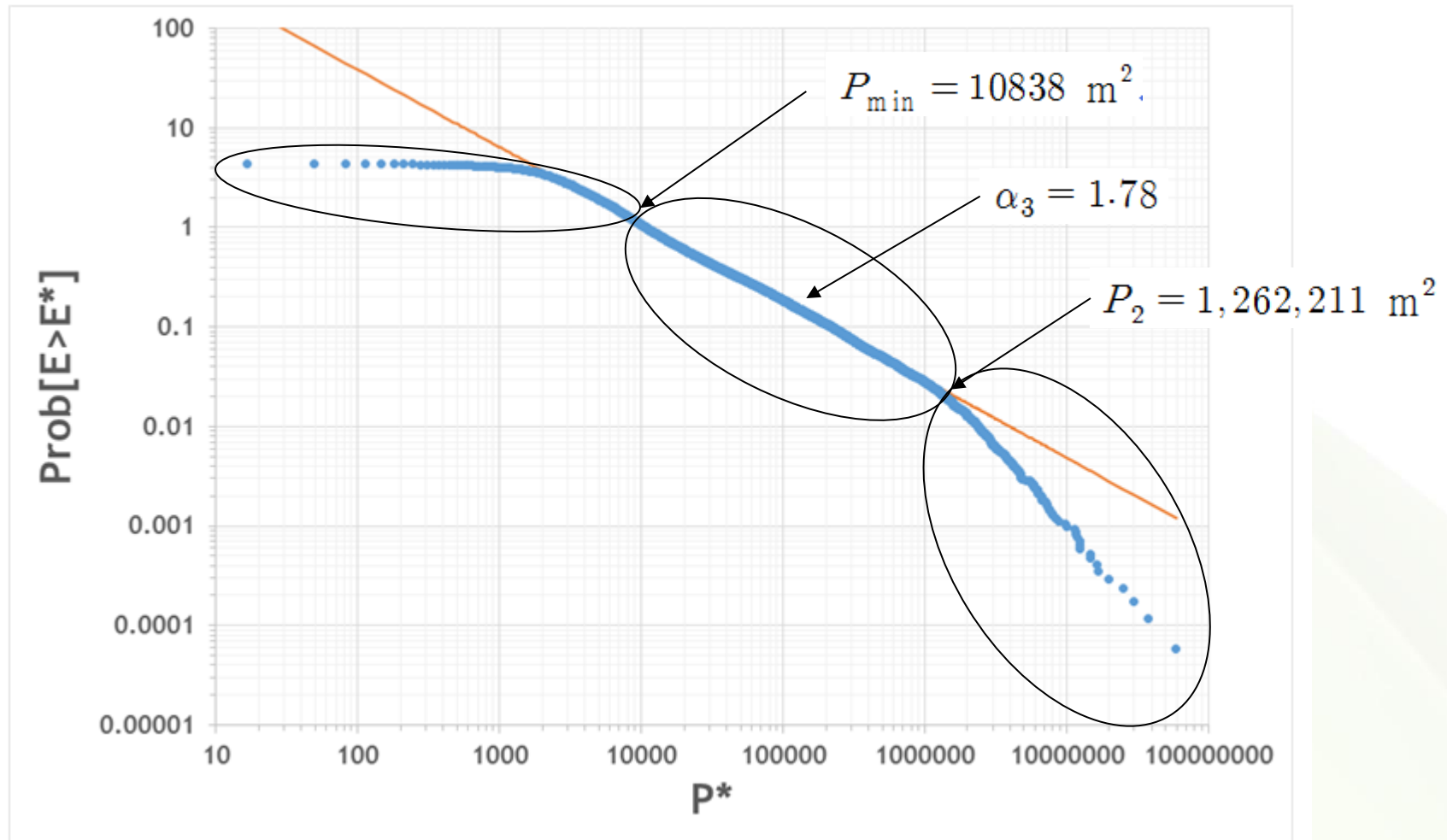
Case Study

▪ Fitting Power Law Distribution by ML



Case Study

▪ Fitting Power Law Distribution of Stream Power



Case Study

In Summary

- Shear stress has the exponent value greater than 2.
- ❖ *Scale dependent property not following power law distribution.*
- Drainage Area and Stream Power have the exponent value less than 2.
- ❖ *Scale independent property following power law distribution.*
- **There are inflection points at both sides of the linear segment.**
- ❖ *Geomorphologic thresholds for soil loss assessment*

Case Study

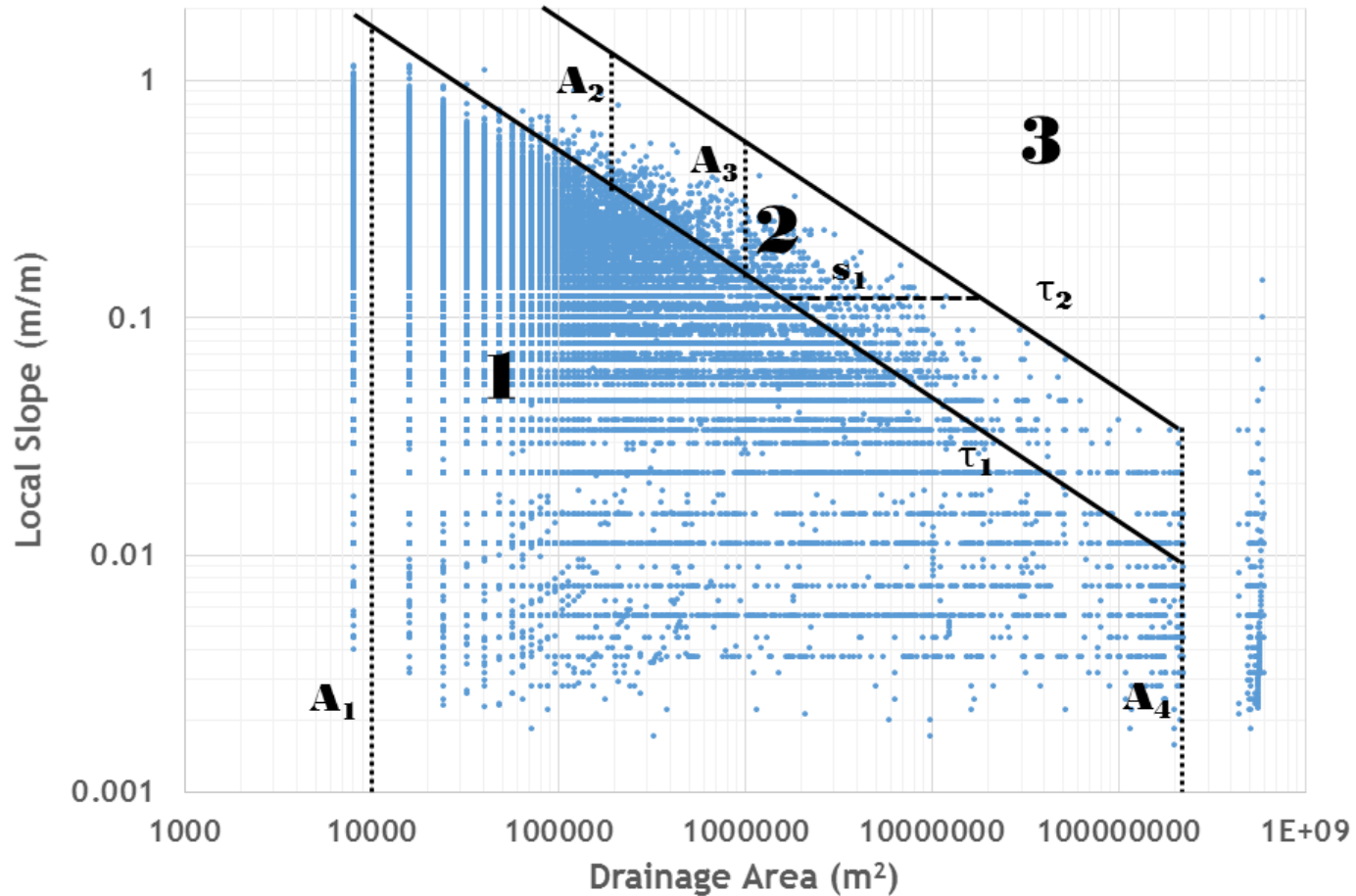
▪ Division of Slope and Area Field

Geomorphologic Threshold of Local Slope–Drainage Area Space

<i>EID</i>	τ_1	158 m
	τ_2	609 m
	P_1	10,838 m ²
	P_2	1,262,211 m ²
<i>s</i>	s_1	0.1213
<i>A</i>	A_1	10,000 m ²
	A_2	184,275 m ²
	A_3	1,016,550 m ²
	A_4	223,009,136 m ²

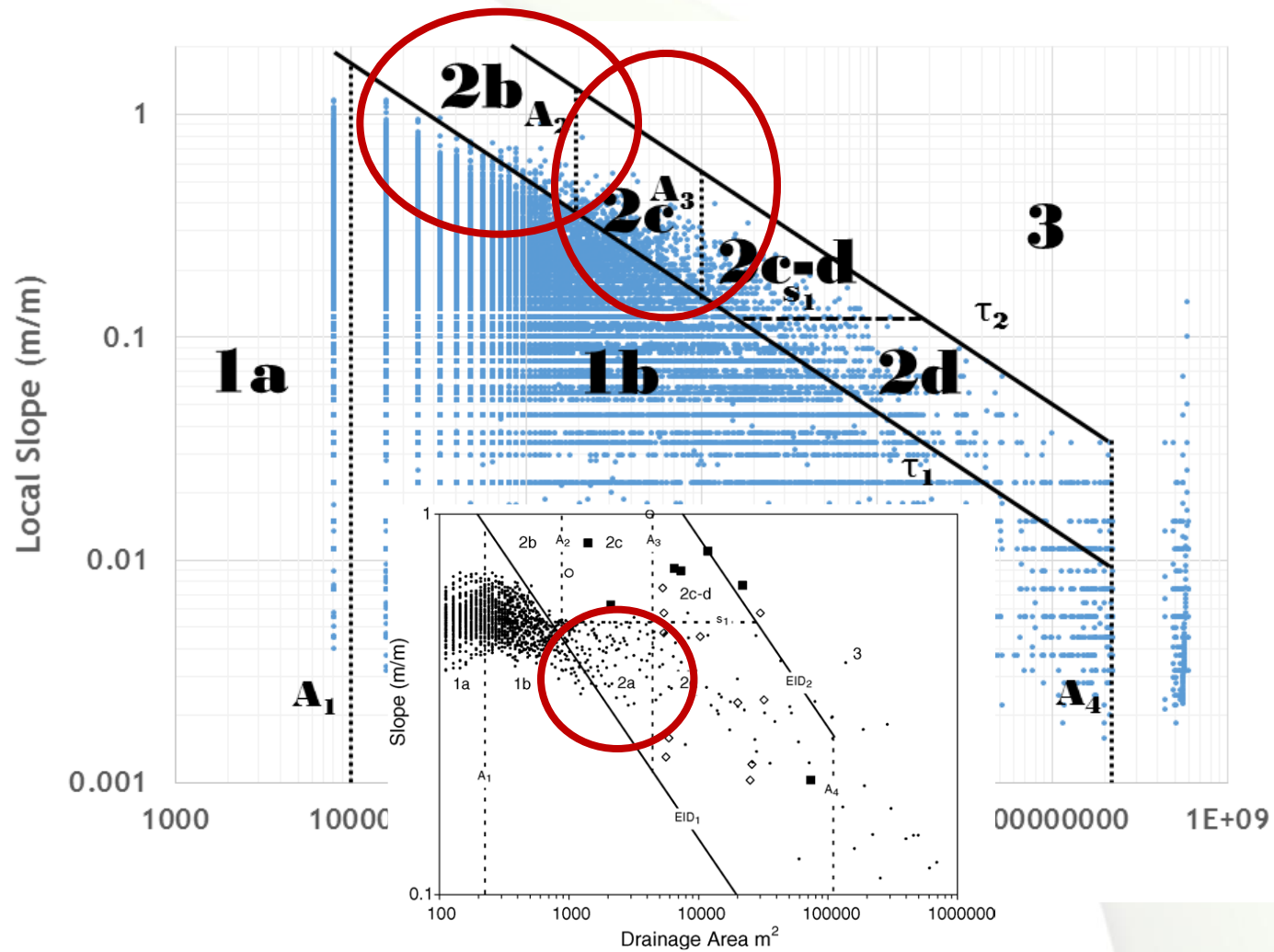
Case Study

- Division of Slope and Area Field
- Shear Stress as Energy Index



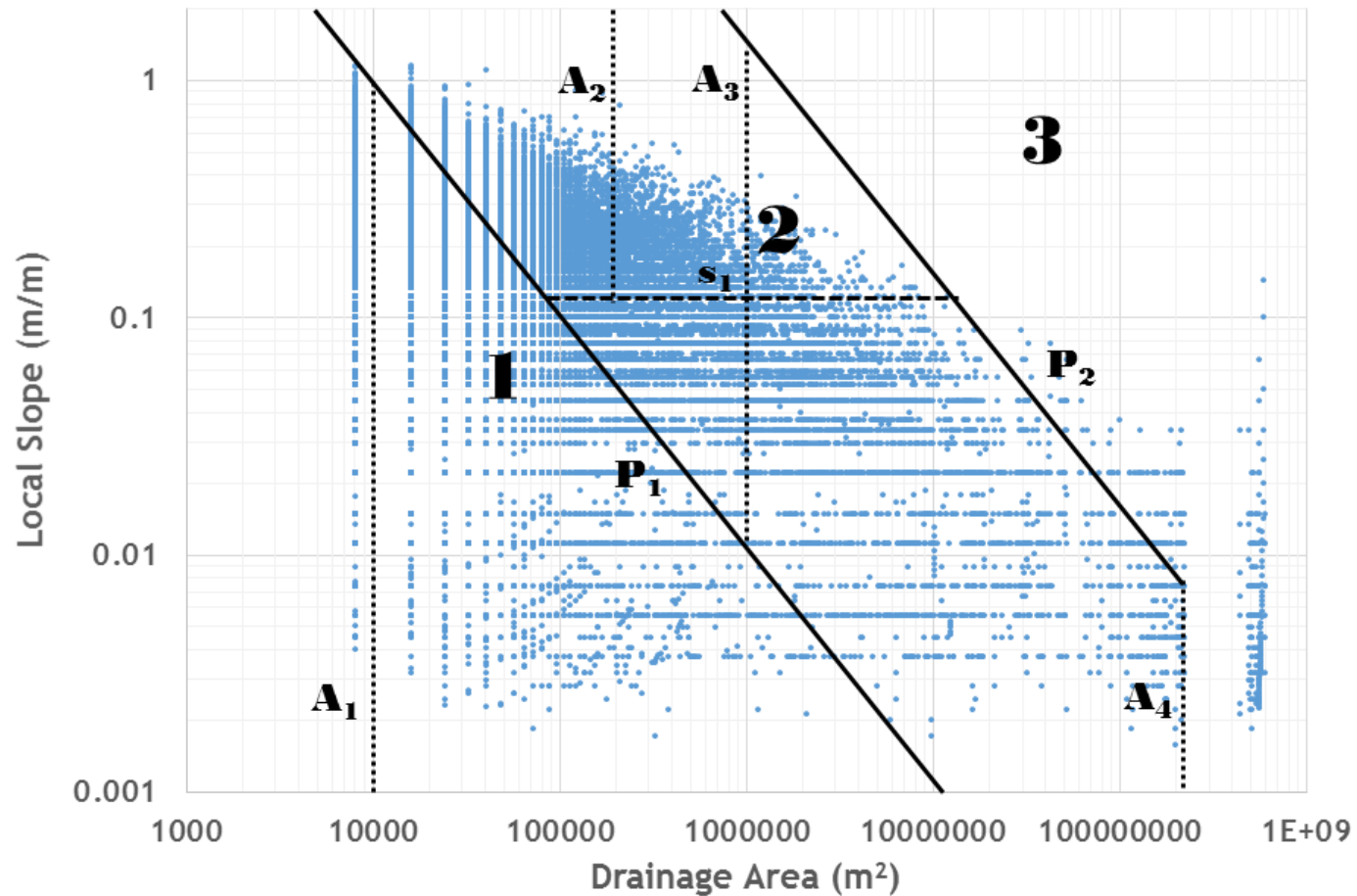
Case Study

- Division of Slope and Area Field
- Shear Stress as Energy Index



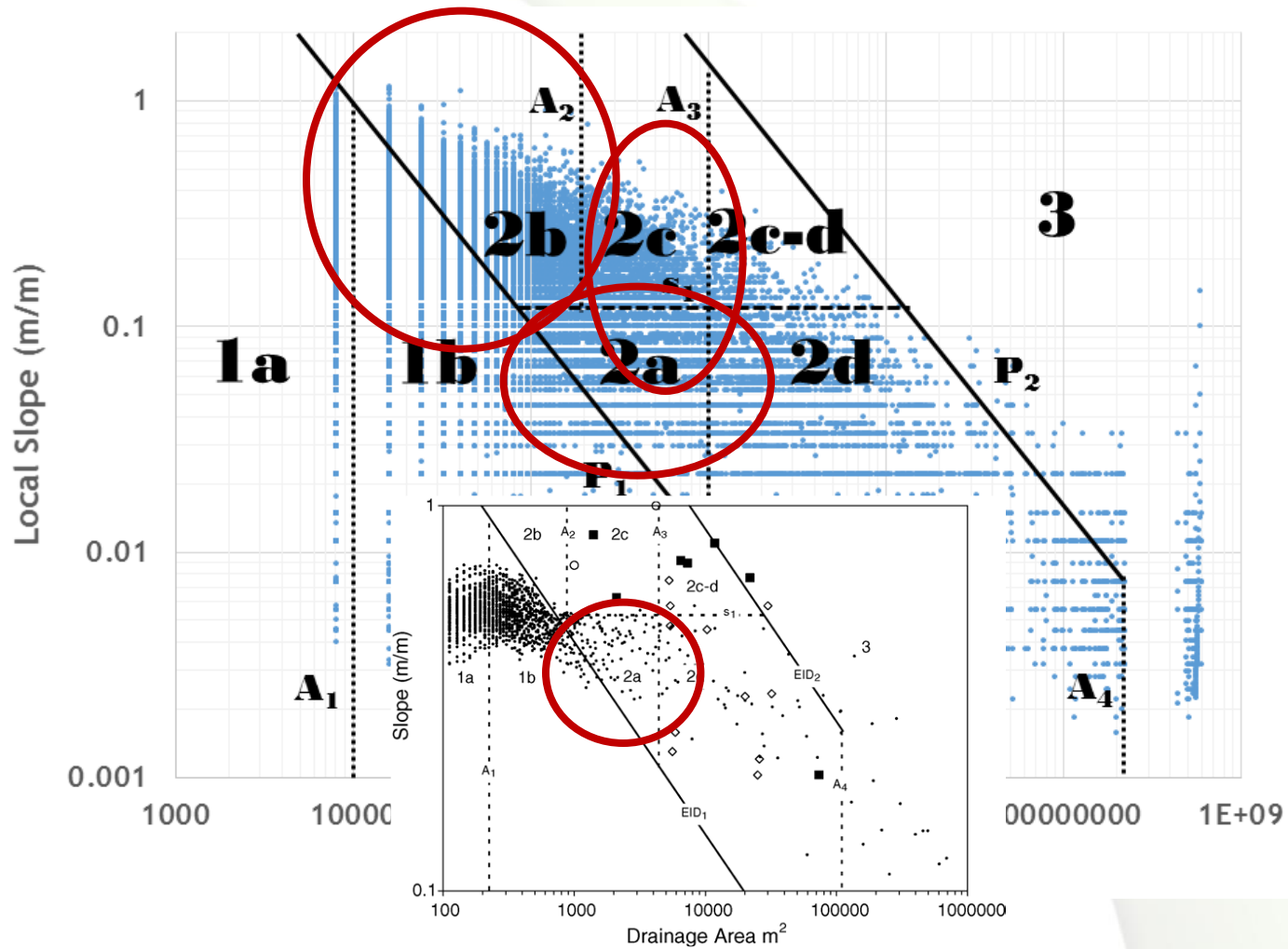
Case Study

- Division of Slope and Area Field
- Stream Power as Energy Index



Case Study

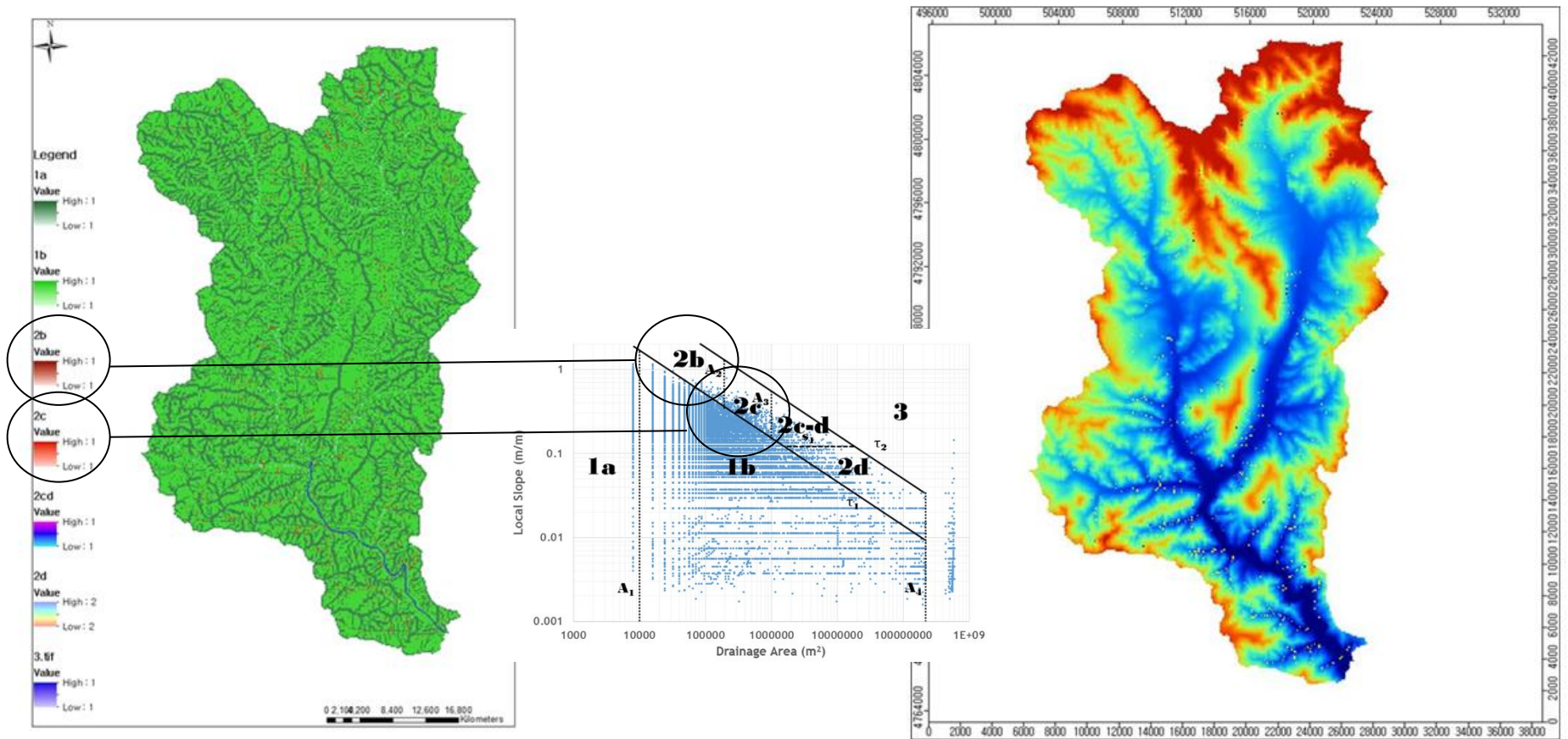
- Division of Slope and Area Field
- Stream Power as Energy Index



Case Study

2-D Plot of Unstable Points

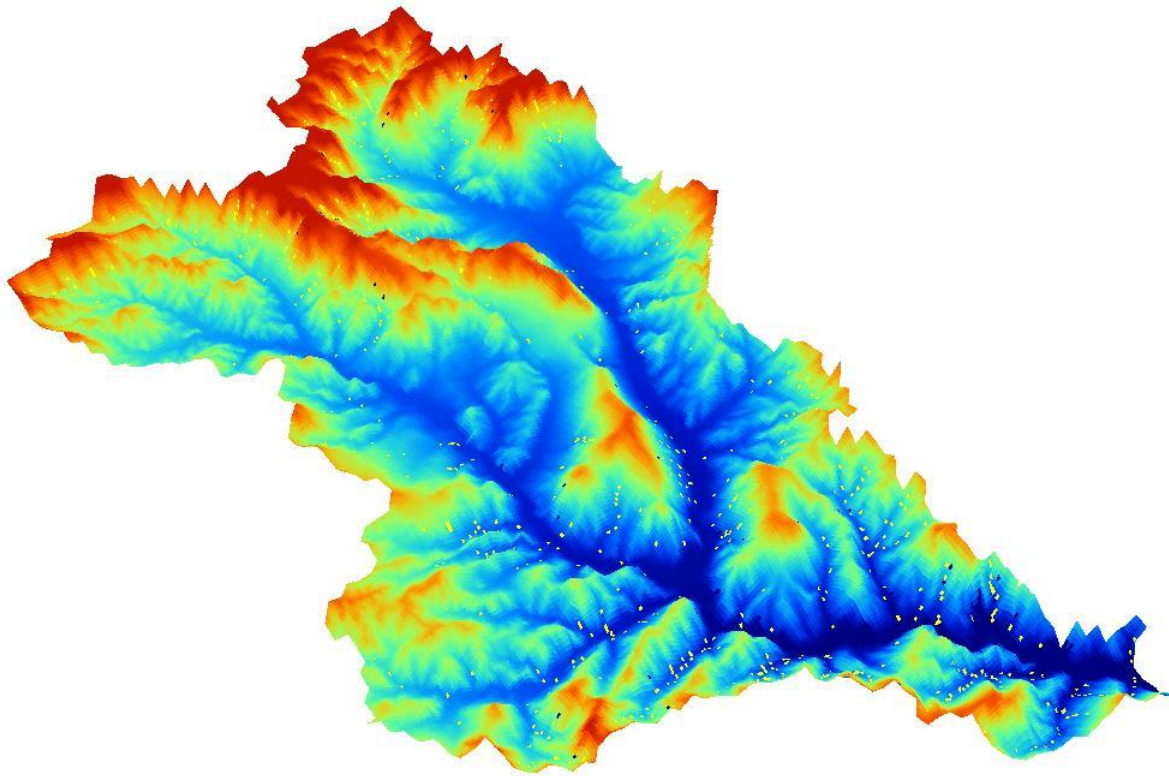
➤ Shear Stress as Energy Index



Case Study

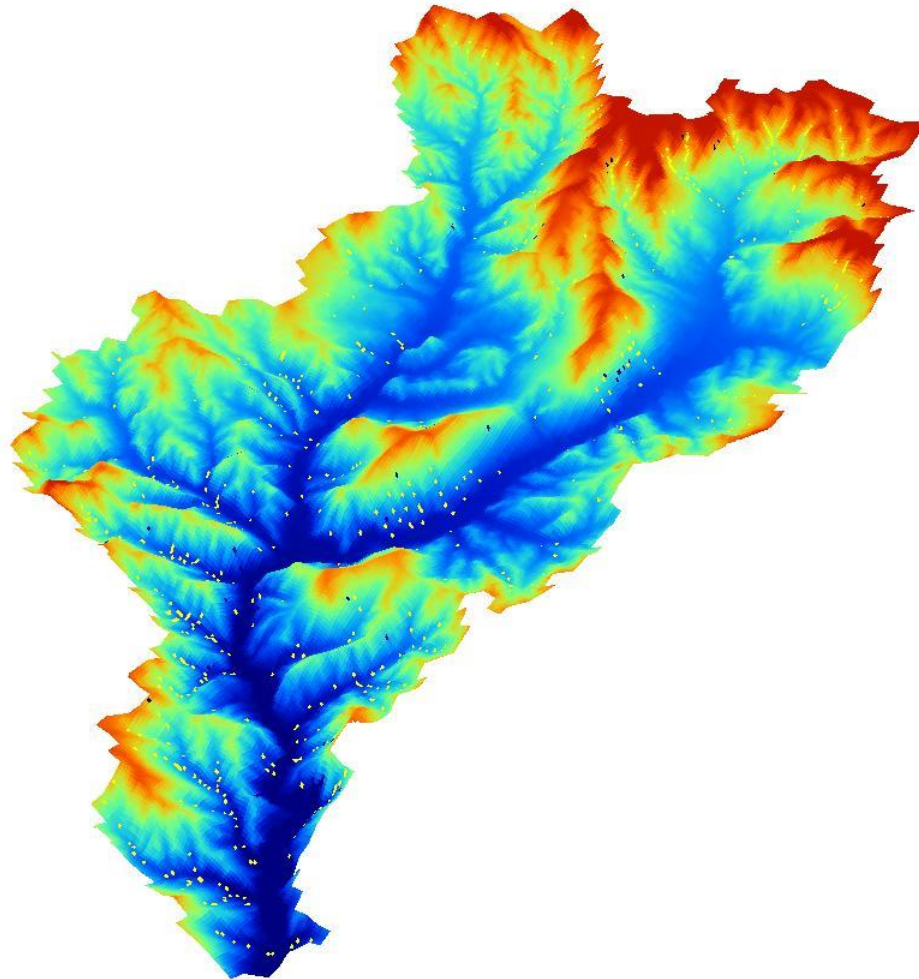
- **3-D Plot of Unstable Points**

- Shear Stress as Energy Index



Case Study

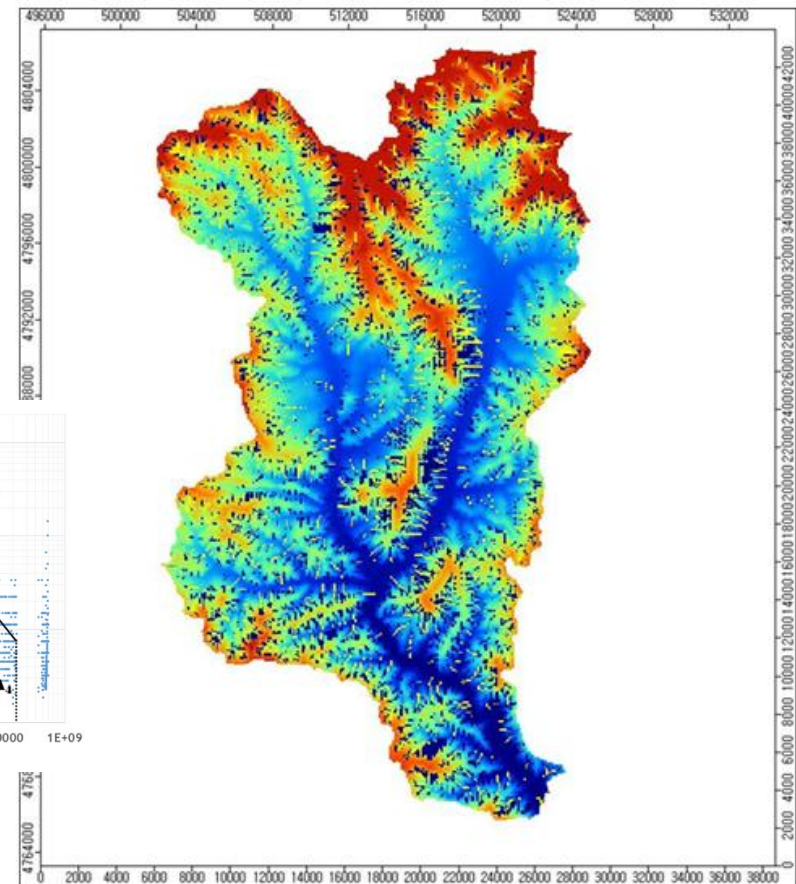
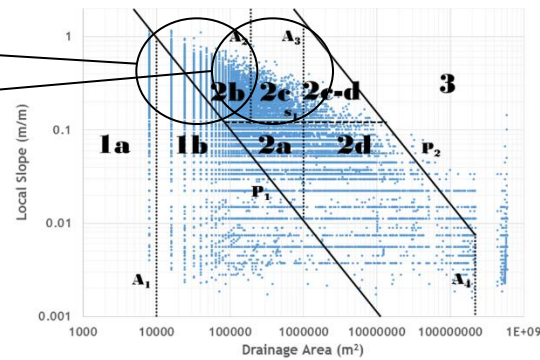
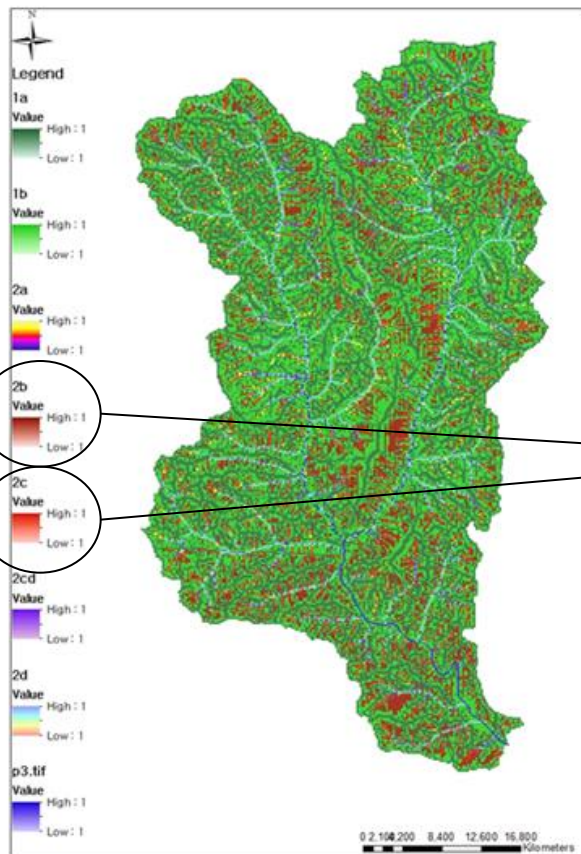
- **3-D Plot of Unstable Points**
- Shear Stress as Energy Index



Case Study

2-D Plot of Unstable Points

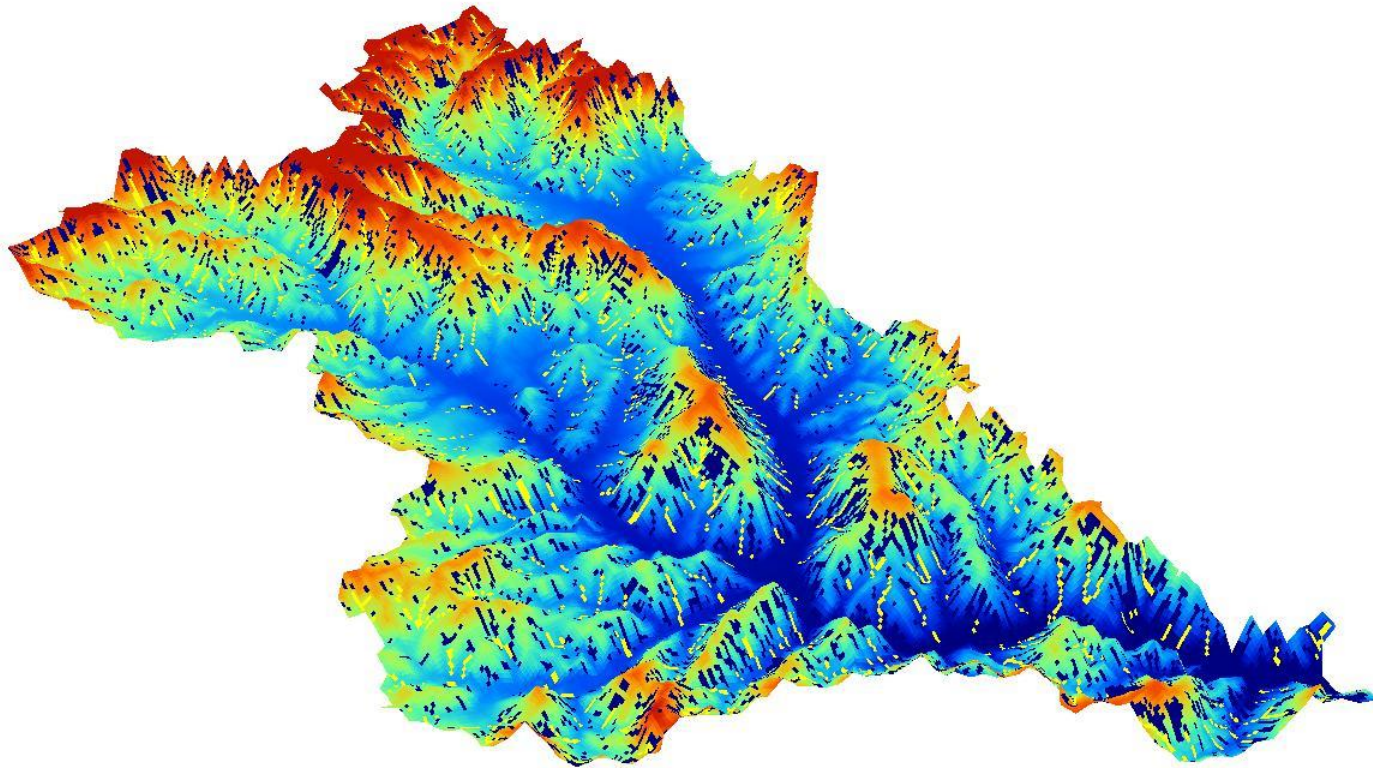
Stream Power as Energy Index



Case Study

- **3-D Plot of Unstable Points**

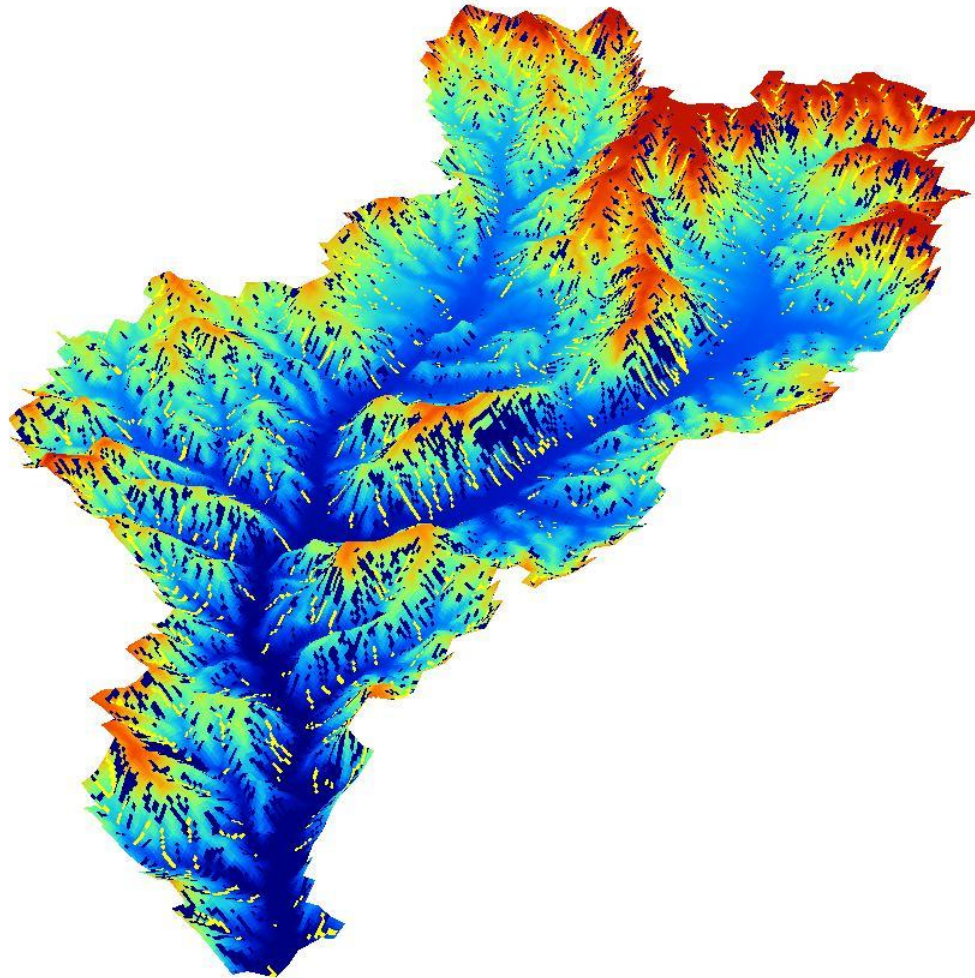
- Stream Power as Energy Index



Case Study

- **3-D Plot of Unstable Points**

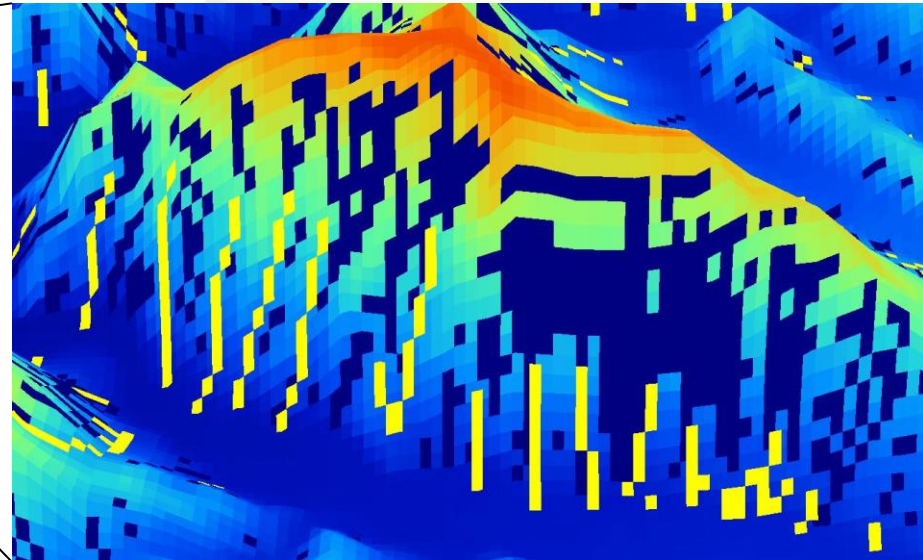
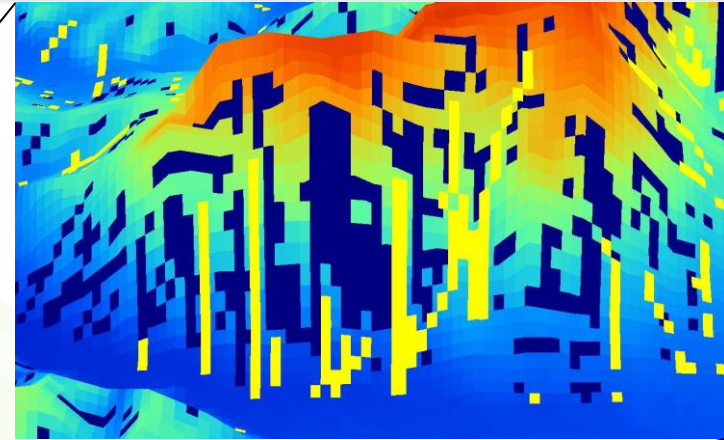
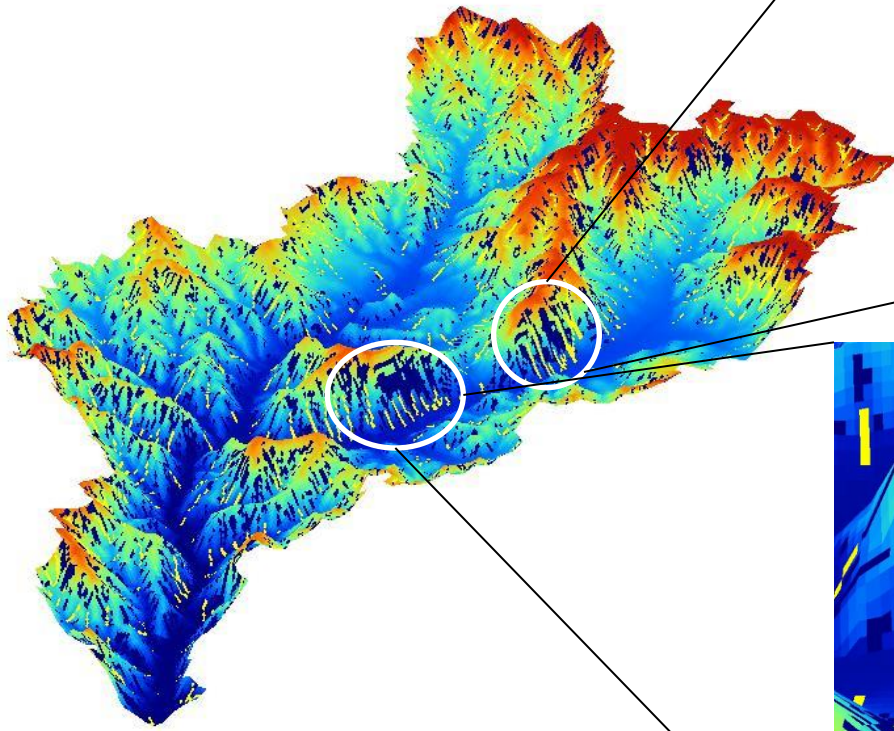
- Stream Power as Energy Index



Case Study

- **3-D Plot of Unstable Points**

- Stream Power as Energy Index



Conclusion

- Three distinct behavioral regimes are observed from the complementary cumulative distributions of three geomorphologic factors.
- Drainage area and stream power can be judged as scale invariance factor without finite scale while tractive force as scale dependence factor with finite scale.
- Furthermore, it is judged that tractive force would not follow power law distribution because it shows limited complex system behaviors only within the small extent of scale.
- It is observed that the range of vulnerable region by scaling regime of tractive force is much narrower than by scaling regime of stream power.
- This is due to the tractive force is a scale dependent factor which does not follow power law distribution and does not adequately reflect energy expenditure pattern of river basins.
- Therefore, stream power is preferred to be a more reasonable factor for the evaluation of soil losses.

Thank you !

